

# Heterogeneity of Agents in Cellular Evacuation Model Explains the Decreasing Bottleneck Flow

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**Abstract** Heterogeneous crowd consisting of pedestrians with essentially diverse abilities behaves in certain aspects differently than a homogeneous crowd consisting of “average” pedestrians. This study investigates the influence of heterogeneity in aspects connected to the ability to navigate through a crowd in front of a bottleneck. Simulations of cellular multi-agent model suggest that the heterogeneity in ability to push through the crowd (represented by aggressiveness) and willingness to bypass the crowd (represented by sensitivity to occupation) may be responsible for the bottleneck flow decreasing in time – a phenomenon observed in experiments.

**Keywords** Pedestrian flow · floor-field model · agents’ heterogeneity · bonds · aggressiveness

## 1 Introduction

Trying to investigate and estimate bottleneck flow from experimental data, it is necessary to identify the steady state part of the evacuation process from temporal evolution of density, flow, or velocity at the bottleneck [1]. This may be a challenging task, especially owing to the fact that in some experimental studies the flow continuously decreases not reaching a desired steady-state plateau [2]. Several explanations of this flow decrease were offered including loss of motivation, or decreasing pressure from behind. This study offers an additional aspect contributing to the explanation of this phenomena – the crowd heterogeneity.

In experimental study [3] an internal heterogeneity related to the ability to navigate through a crowd in front of a bottleneck was identified in a traditionally homogeneous

group of university students: ability to push through the crowd and willingness to bypass the crowd. The study [4] aimed to mimic such heterogeneity in modified cellular floor-field model by introducing the principle of bonds and aggressiveness parameter. The floor-field cellular model [5] belongs to the most considered cellular models of pedestrian dynamics and there exist a lot of extensions and modifications [6, 7]. Recent studies [8, 9] focus on above mentioned internal heterogeneity related to the aggressiveness of considered agents.

The presented study leans over the theses [10, 11], which can serve in certain sense as supplementary material to this paper.

## 2 Model Description

We consider a floor-field based cellular model investigated in [4], i.e static-field based choice of target cells enriched by the principle of bonds connected to possibility of choosing an occupied target cell, aggressiveness parameter generalizing the friction-based conflict solution, and heterogeneity in aggressiveness and sensitivity to occupation leading to various strategies of pushing through or bypassing the crowd. This section briefly summarizes the model description from [4, 12, 13] with emphasis on improved concept of sensitivity to occupation given by Eq. 1.

Agents are moving along a rectangular lattice  $\mathbb{L} \subset \mathbb{Z}^2$  by means of probabilistic hopping to neighbouring cells from Moore neighbourhood following the exclusion rule. In more detail, an agent occupying cell  $x \in \mathbb{L}$  chooses its new target cell  $y \in N(x) = \{z \in \mathbb{L}; |x_1 - z_1|, |x_2 - z_2| \leq 1\}$  with probability  $P(x \rightarrow y)$ . The probability  $P(x \rightarrow y)$  is influenced by static field  $S$ , occupation indicator  $O$ , and diagonal motion indicator  $D$ , accompanied by sensitivity parameters  $k_S \in [0, +\infty)$ ,  $k_O \in [0, 1]$ , and  $k_D \in [0, 1]$  respectively. Specifically,  $S(y)$  is equal to minimum steps required to reach the exit cell  $e$  from cell  $y$ ,  $D(x, y) = \mathbf{1}_{\{(x_1 - y_1)(x_2 - y_2) \neq 1\}}$  is the identifier of diagonal movement, and  $O(y|x) = \mathbf{1}_{\{x=y \vee y \text{ empty}\}}$  is the identifier of unoccupied cells.

Compared to the original model an improved target choice probability is introduced in this paper. The probability

$$P(x \rightarrow y) = k_O \cdot P_O(x \rightarrow y) + (1 - k_O) \cdot P_S(x \rightarrow y) \quad (1)$$

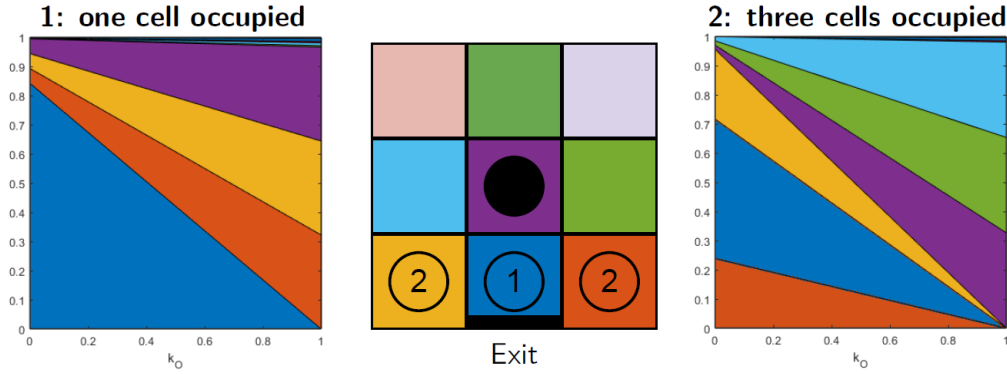
is defined as probabilistic mixture with parameter  $k_O$  as mixing parameter. The distribution

$$P_O(x \rightarrow y) \propto \exp\{-k_S S(y)\} \cdot O(y|x) \cdot (1 - k_D D(x, y)) \quad (2)$$

respects the exclusion rule (occupied cell is never chosen), whereas the distribution

$$P_S(x \rightarrow y) \propto \exp\{-k_S S(y)\} \cdot (1 - k_D D(x, y)) \quad (3)$$

ignores the occupation of neighbouring cells (agent hopes to enter the cell by means of the bonds principle, see below). The dependence of the target choice distribution on parameter  $k_O$  for two situations is depicted in Fig. 1.



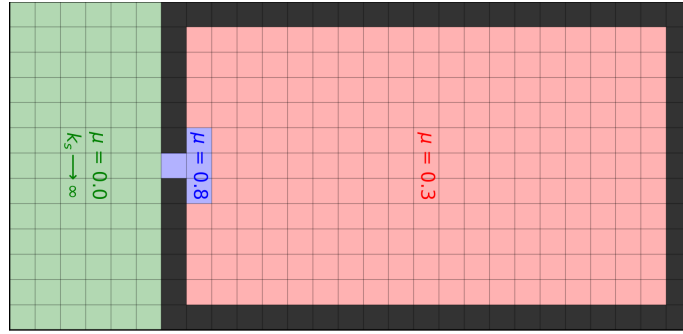
**Figure 1** Stacked graphs illustrating the target choice distribution Eq. 1 with respect to  $k_O$  for two cases. 1: one cell in the exit direction occupied, others empty. 2: three cells in the exit direction occupied, others empty.  $k_S = 3.5$ ,  $k_D = 0.5$ .

If an agent chooses an occupied cell, a bond to the cell is created, which lasts to the next actualization of the agent. If the target cell is or becomes free during the actual step, the bonded agent tries to enter it within this step (and may compete with other agents, see conflicts below). This means that the bonds during each step form a set of trees with roots in empty target cells; the update then proceed in backward-sequential order from the roots to the leaves. As the trees do not intersect (an agent chooses exactly one target cell), the process can be easily parallelized. The model is constructed to enable partially asynchronous update using adaptive time span [12] – every agent can have its own updating frequency emulating variations in speed. This asynchronous concept, however, appears to be superfluous, since the stochastic nature of the motion provides sufficient variance of average speed [14]. In the current paper, the partial asynchronism is caused solely by diagonal motion, which takes  $3/2 \approx \sqrt{2}$  longer time. The time  $t$  is therefore measured in algorithm steps, i.e.  $t \in T = \{0, 1, \dots\}$ .

Parallel (or partially synchronous) update brings conflicts, i.e. situations in which two or more agents choose the same target cell. Commonly used concepts are based on friction [15]: with certain probability  $\mu \in [0, 1]$  the conflicts blocks the motion and none of the agents move; with complementary probability, the conflict resolves to one of conflicting agents to be chosen randomly to enter the target cell (winning the conflict), others do not move. In [4] an additional parameter affecting the conflict solution has been added, namely the aggressiveness parameter  $\gamma \in [0, 1]$ . Aggressiveness is considered to be dedicated agent-parameter expressing the agents ability to win conflicts. From agents in  $\{x_1, \dots, x_k\}$  attempting to enter the same target cell, such agents from  $x_j$  is chosen with  $j = \operatorname{argmax}_{i=1, \dots, k} \gamma_i$ . If no other agent has the same aggressiveness, agent in  $x_j$  wins the conflict immediately. If there are two or more agents with the same highest aggressiveness a parameter of friction  $\mu$  affects the conflict. With probability  $\mu(1 - \gamma_j)$  the conflict blocks the motion. With complementary probability, one of the agents with maximal aggressiveness is chosen at random. The model used in this simulation study incorporates the spatially dependent friction introduced in [13], i.e. the friction parameter  $\mu$  has different value in the room, near the exit, and behind the exit, as indicated by Fig. 2.

### 3 Simulations

For purposes of this study, the evacuation of a rather small rectangular room with the exit in the middle of one side was simulated by means of above described model. An example of the cellular representation of the room is depicted in Fig. 2. The room sizes  $19 \times 11$  cells and  $15 \times 15$  cells were used. Initial positions of the  $N = 70$  agents were randomly selected in the third of the room opposite to the exit. The simulation was run  $n = 500$  times for each setting in order to obtain representative set of trajectories of the underlying random process. Conclusions drawn in this study were based on aggregated information from these  $n$  trajectories per setting.



**Figure 2** Schematic representation of the  $19 \times 11$  cells large room used for the simulation.

Results presented in this study are based on simulations with following parameters:  $k_S = 3.5$ ,  $\mu_{\text{exit}} = 0.8$ ,  $\mu = 0.3$ ,  $k_D = 0.7$ . The parameters are ordered according to their impact on Total Evacuation Time (TET), which was previously studied in [10, 11]. The aggressiveness parameter  $\gamma \in [0, 1]$  and sensitivity to occupation  $k_O \in [0, 1]$  are parameters of interest in this study. By heterogeneity in aggressiveness we further understand that the agent's aggressiveness is chosen from two values  $\gamma_1 < \gamma_2$ , homogeneity in aggressiveness means that all agents have equal aggressiveness  $\gamma$ . Similarly, the heterogeneity in occupation is expressed by  $k_{O1} < k_{O2}$ , and homogeneity by single value  $k_O$ . In both heterogeneous cases the ratio of agents with given parameter values is 1 : 1.

Next to the evacuation time, this study investigates the flow through the bottleneck – the exit. Let us denote by  $X_{t,i}(\omega)$  the number of pedestrians passing through the exit cell(s) at time  $t \in T$  in the  $i$ -th trajectory/simulation ( $i \in \{1, 2, \dots, n\}$ ) under the realization  $\omega$ . The flow at time  $t$  was approximated by sample average of values obtained from the simulations, i.e

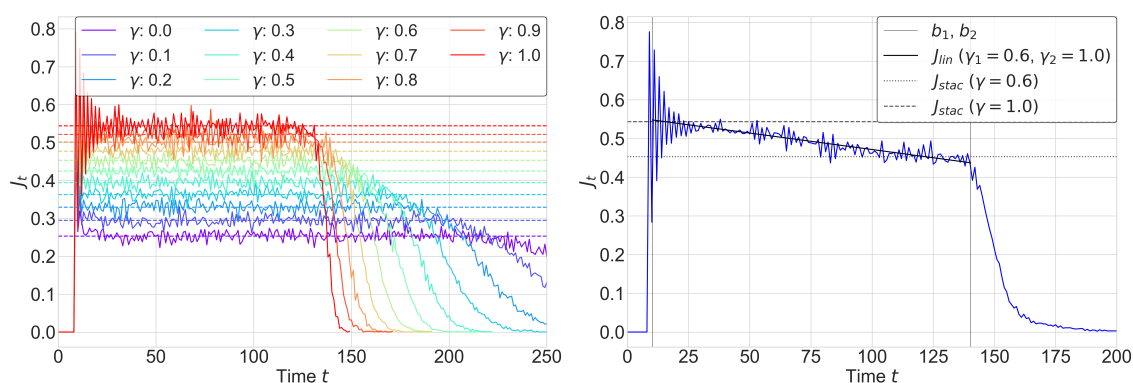
$$J_t = \mathbb{E}X_{t,i} \approx \frac{1}{n} \sum_{i=1}^n X_{t,i}(\omega). \quad (4)$$

The bottleneck flow  $J_t$  is assumed to be close to piece-wise linear function in  $t$  with following segments: 1)  $J_t = 0$  before first agents reach exit, 2) sharp increase of  $J_t$  up-to the steady-flow part, 3) steady flow  $J_t = J_{\text{stac}}$ , 4) sharp decrease of  $J_t$  as crowd in front of the bottleneck shrinks, 5)  $J_t = 0$  after last agents evacuate. The breakpoint positions of this model were computed using the Muggeo's iterative algorithm [16] from piecewise-

regression python-package [17]. Main focus is on break-points  $b_1, b_2$  defining the stationary part 3).

## 4 Heterogeneity in parameters

Considering a homogeneous setting with one  $k_O$  and one  $\gamma$  for all agents, the steady-state flow  $J_{\text{stac}}$  can be determined by means of above mentioned model, as illustrated by left graph in Fig. 3. However, considering **heterogeneity in aggressiveness** with  $\gamma_1 < \gamma_2$ , the stationary part of the flow cannot be described as constant function  $J_t = J_{\text{stac}}$ , but rather a linearly decreasing function  $J_t = J_{\text{stac}} = \beta_1 \cdot t + \beta_0$  with coefficient  $\beta_1 < 0$  closely related to the difference  $\gamma_2 - \gamma_1$ . An example of such flow dependence is illustrated in right graph in Fig. 3 and further graphs in [11, Appendix B].

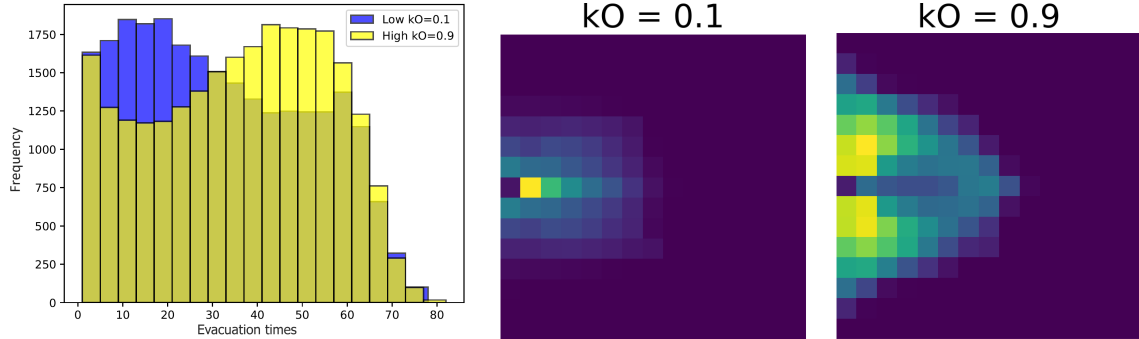


**Figure 3** Effect of heterogeneity in  $\gamma$ . Left: the steady flow part for homogeneous group of agents can be approximated by a constant function  $J_t = J_{\text{stac}}$ ; the flow  $J_{\text{stac}}$  increases with increasing  $\gamma$ . Right: In heterogeneous group of agents the stationary part can be approximated by linearly decreasing function  $J_t$ .

The minimal value of this linear part is equal to the stationary flow for homogeneous group with  $\gamma = \gamma_1$ . The maximal value of the linear part is equal to the stationary flow for homogeneous group only for  $\gamma_2 = 1$  and is lower otherwise. The explanation of the decreasing flow phenomenon lies in the definition of the conflict-solution mechanism. As the agents with  $\gamma_2$  are more successful in the conflicts, they have higher probability to reach and pass the exit and thus leave the room earlier. Near the end of the simulation, the crowd consists mainly of agents with lower  $\gamma_1$ . Because the probability of the conflict to be blocked due to the friction increases with decreasing aggressiveness  $\gamma$ , higher ratio of  $\gamma_1$ -agents in the crowd results in lower flow. The linear decrease of the bottleneck flow was as well identified in simulations with heterogeneous group of agents consisting of more agent types with more values of aggressiveness  $\gamma$ .

The above described linear decrease of the flow was observed even when considering **heterogeneity in occupation**  $k_O$ . The explanation is similar: Lower  $k_O$  means that agent in dense crowd is more likely to create a bond and move towards the exit in line, comparing to an agent with higher  $k_O$ , which is forced to choose his current cell as its target

cell. This effect is illustrated by the histogram in the most left graph in Fig. 4. Evacuation times (i.e. time-steps taken by the agents to reach the exit), of 70 agents with heterogeneous occupancy  $k_O \in \{0.1; 0.9\}$  were aggregated over 500 simulations. We can see that the proportion of agent with low  $k_{O1} = 0.1$  is higher shortly after the beginning of the evacuation and lower at the end of the evacuation.



**Figure 4** Heterogeneity in  $k_O$ . Left: frequency of evacuation times aggregated over agents' occupancy  $k_O$ . Right: Heat-maps of trajectory counts passing through given cell aggregated over agents' occupancy  $k_O$ .

Dominating effect of the heterogeneity in occupancy consists in different path choice in front of the bottleneck. As can be seen from the trajectory heat-maps in Fig. 4, the agents with low  $k_O = 0.1$  dominantly stayed and moved in the line in front of the exit, while the agents with high  $k_O = 0.9$  tried to bypass the crowd from left or right. From this point the parameter  $k_O$  enables to mimic the bypassing strategy observed in [3]. However, due to the cellular structure of the space in front of the bottleneck, the bypassing strategy does not allow the agents to squeeze along the walls as observed in the experiments and thus does not have the accelerating effect, as expected.

## 5 Conclusions

The floor-field model with bonds principle and aggressiveness from [4] was improved by introducing the target choice probability  $P(x \rightarrow y)$  as a probabilistic mixture Eq. 1 with mixing parameter  $k_O$ . A simulation study focusing on the effect of the heterogeneity in aggressiveness  $\gamma$  and occupation sensitivity  $k_O$  was performed. This study shows that such heterogeneity can partially mimic the internal heterogeneity in navigation through the crowd identified in [3]. More important, the study suggests that the heterogeneity in aggressiveness or sensitivity to occupation may be one of the factors explaining the decreasing bottleneck flow during the emptying of the room. However a detailed analysis of the experimental data is needed to confirm or reject such hypothesis drawn from a simulation study using one specific cellular model.

**Author Contributions** Pavel Hrabák: Conceptualization, Methodology, Writing – review and editing, Supervision / Matej Šutý: Software, Formal analysis, Writing – original draft / Mykola Hotlib: Software, Formal analysis.

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